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The transmission of a plane acoustic wave through a non-uniform thermoelastic layer☆

N.V. Larin, L.A. Tolokonnikov

Tula, Russia

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Abstract

The reflection and refraction of a plane acoustic wave by a thermoelastic plane layer, non-uniform in thickness, bounded by non-viscous heat-conducting liquids, generally different, is considered. The system of equations for small perturbations of the thermoelastic medium is reduced to a system of ordinary differential equations, the boundary-value problem for which is solved by two methods: the spline-collocation method and the power-series method. Analytic expressions are obtained which describe the wave fields outside the layer. The results of calculations of the intensity transmission coefficient of the acoustic wave are presented.

The reflection and refraction of a plane acoustic wave by a non-uniform isotropic plane layer was investigated previously in Ref. 1. A solution of the problem of the transmission of a plane monochromatic acoustic wave through a transversely isotropic non-uniform plane layer was obtained in Ref. 2. The transmission of sound through a non-uniform anisotropic layer, bounded by viscous liquids was investigated in Ref. 3. The solution of the problem of the reflection and refraction of an acoustic wave by a plane non-uniform layer, the material of which possesses anisotropy of general form, was obtained in Ref. 4. In all the above investigations, thermal processes in the elastic non-uniform layers were ignored.

1. Formulation of the problem

Consider a non-uniform isotropic thermoelastic plane layer of thickness 2H, having a constant temperature T_0 in the unperturbed state. There are no heat sources in the layer. The system of rectangular coordinates x_1, x_2, x_3 is chosen in such a way that the x_1 axis lies in the middle plane of the layer, while the x_3 axis is directed downwards along the normal to the layer surface. The moduli of elasticity, the temperature coefficient of linear expansion and the thermal conductivity of the material of the layer are described by differentiable functions of the coordinate x_3. The density of the layer material and its volume heat capacity are described by continuous functions of the x_3 coordinate. We will assume that the lower and upper surfaces of the layer are bounded by non-viscous heat-conducting uniform liquids, which have a temperature T_0, densities ρ_1 and ρ_2 and velocities of sound c_1 and c_2 respectively.

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E-mail address: tolla@tula.net (N.V. Larin).

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Suppose a plane acoustic wave, the velocity potential of which is

$$\Psi_i = A_i \exp\{i[k_{21}^1 x_1 + k_{21}^3 (x_3 + H) - \omega t]\} \tag{1.1}$$

is incident on the thermoelastic layer from the half-space $x_3 < -H$, where A_i is the amplitude of the incident wave, $k_{21}^1 = k_{21}\sin\theta_1$ and $k_{21}^3 = k_{21}\cos\theta_1$ are the projections of the wave vector $\mathbf{k}_{21}$ onto the x_1 and x_3 axes respectively, k_{21} is the wave number of the acoustic wave in the upper half-space, ω is the angular frequency and θ_1 is the angle of incidence of the plane wave. The time factor $\exp(-i\omega t)$ is henceforth omitted.

We will determine the waves reflected from the layer and transmitted through it, and we will also find the displacement and temperature fields in the thermoelastic layer.

2. The wave-field equations

The propagation of thermoelastic waves in a non-uniform isotropic layer is described by the general equations of motion of a continuum[5]

$$\frac{\partial \sigma_{ij}}{\partial x_j} = \rho \ddot{u}_i, \quad i = 1, 3 \tag{2.1}$$

and the heat flux equation[6]

$$\frac{\partial}{\partial x_i}\left(\lambda_T \frac{\partial T}{\partial x_i}\right) - \gamma \operatorname{div}\dot{\mathbf{u}} = c_v \dot{T} \tag{2.2}$$

where σ_{ij} are the stress tensor components, which are related to the strain tensor components ε_{ij} and the change in temperature T of the perturbed layer by the Duhamel-Neumann relations[6]

$$\sigma_{ij} = 2\mu\varepsilon^{ij} + (\lambda \operatorname{div}\mathbf{u} - \beta T)\delta_{ij}; \quad \varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right), \quad \operatorname{div}\mathbf{u} = \varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33} \tag{2.3}$$

u_i is the projection of the displacement vector $\mathbf{u}$ onto the x_i axis, $\rho = \rho(x_3)$ is the density of the layer material, $\lambda = \lambda(x_3)$, $\mu = \mu(x_3)$ are the moduli of elasticity of the layer material, $\beta = \beta(x_3) = 3\alpha_T K$, $\alpha_T = \alpha_T(x_3)$ is the temperature coefficient of linear expansion of the layer material, $K = \lambda + (2/3)\mu$ is the isothermal modulus of volume expansion, $\lambda_T = \lambda_T(x_3)$ and $c_v = c_v(x_3)$ are the thermal conductivity and volume heat capacity of the layer material respectively, $\gamma = T_0\beta$ and δ_{ij} is the Kronecker delta.

Since the wave vector of the incident wave lies in the x_1, x_3 plane and, consequently, the exciting field is independent of the x_2 coordinate, while the nonuniformity of the layer material only manifests itself along the x_3 axis, neither the reflection nor the transmission in the half-space $x_3 > H$ nor the field excited in the thermoelastic layer should depend on the x_2 coordinate. Since the layer is isotropic, the component u_2 of the displacement vector $\mathbf{u}$ is equal to zero. Note also that, by Snell's law,[7] the dependence of the other two components of the displacement vector and the temperature increment on the x_1 coordinate will have the form $\exp(ik_{21}^1 x_1)$, and hence the projections of the displacement vector and the temperature increment will be sought in the form

$$u_r = U_r(x_3)\exp(ik_{21}^1 x_1), \quad r = 1, 3; \quad T = \tilde{T}(x_3)\exp(ik_{21}^1 x_1) \tag{2.4}$$

We will introduce the following dimensionless quantities

$$x = \frac{x_3}{H}, \quad U_1^* = \frac{U_1}{H}, \quad U_3^* = \frac{U_3}{H}, \quad T^* = \frac{\tilde{T}}{T_0}, \quad \lambda^* = \frac{\lambda}{\lambda_0}$$

$$\mu^* = \frac{\mu}{\mu_0}, \quad \rho^* = \frac{\rho}{\rho_0}, \quad \alpha_T^* = \frac{\alpha_T}{\alpha_T^0}, \quad \lambda_T^* = \frac{\lambda_T}{\lambda_T^0}, \quad c_v^* = \frac{c_v}{c_v^0}$$

Here λ_0, μ_0, ρ_0, α_T^0, λ_T^0, c_v^0 are characteristic quantities.

Substituting expressions (2.3) and (2.4) into Eqs. (2.1) and (2.2), we obtain a system of second-order linear ordinary differential equations in the unknown functions $U_1^*(x)$, $U_3^*(x)$ and $T^*(x)$

$$A\mathbf{F}'' + B\mathbf{F}' + C\mathbf{F} = 0 \tag{2.5}$$

where

$$\mathbf{F} = (U_1^*, U_3^*, T^*)^T$$

$$A = \operatorname{diag}\{a_{11}, a_{22}, a_{33}\}, \quad B = \|b_{\alpha\beta}\|, \quad C = \|c_{\alpha\beta}\|; \quad \alpha, \beta = 1, 2, 3$$

$$a_{11} = \mu^*, \quad a_{22} = l\lambda^* + 2\mu^*, \quad a_{33} = \lambda_T^*$$

$$b_{11} = \mu^{*\prime}, \quad b_{12} = s_1(l\lambda^* + \mu^*), \quad b_{13} = b_{31} = 0$$

$$b_{21} = s_1(l\lambda^* + \mu^*), \quad b_{22} = l\lambda^{*\prime} + 2\mu^{*\prime}, \quad b_{23} = -l_1\beta^*, \quad b_{32} = s\beta^*, \quad b_{33} = \lambda_T^{*\prime}$$

$$c_{11} = q_0\rho^* + s_1^2(l\lambda^* + 2\mu^*), \quad c_{12} = s_1\mu^{*\prime}, \quad c_{13} = -s_1 l_1\beta^* \tag{2.6}$$

$$c_{21} = s_1 l\lambda^{*\prime}, \quad c_{22} = q_0\rho^* + s_1^2\mu^*, \quad c_{23} = -l_1\beta^{*\prime}$$

$$c_{31} = ss_1\beta^*, \quad c_{32} = 0, \quad c_{33} = q_1 c_v^* + s_1^2\lambda_T^*$$

$$\beta^* = 3\alpha_T^*\left[l\lambda^* + \frac{2}{3}\mu^*\right], \quad l = \frac{\lambda_0}{\mu_0}, \quad l_1 = \alpha_T^0 T_0$$

$$s = i\frac{\omega H^2\alpha_T^0\mu_0}{\lambda_T^0}, \quad s_1 = ik_{21}^1 H, \quad q_0 = \frac{\rho_0\omega^2 H^2}{\mu_0}, \quad q_1 = i\frac{\omega H^2 c_v^0}{\lambda_T^0}$$

The prime denotes a derivative with respect to x.

We will represent the velocity of the particles of the liquid in the lower half-space ($j = 1$) and the upper half-space ($j = 2$) in the form

$$\mathbf{v}_j = \operatorname{grad}(\Psi_j + \Phi_j), \quad j = 1, 2$$

The velocity potentials of the acoustic waves Ψ_j and the thermal waves Φ_j are the solutions of the following equations

$$\Delta\Psi_j + k_{j1}^2\Psi_j = 0, \quad \Delta\Phi_j + k_{j2}^2\Phi_j = 0; \quad j = 1, 2$$

where $\Psi_2 = \Psi_i + \Psi_s$, Ψ_s is the velocity potential of the reflected acoustic wave, and k_{j1} and k_{j2} are the wave numbers of the acoustic and thermal waves respectively. Here

$$k_{jl}^2 = \frac{-M_j - (-1)^l\sqrt{M_j^2 + 4L_jN_j}}{2N_j}, \quad j, l = 1, 2$$

where

$$L_j = \frac{\omega^2}{c_j^2}\gamma_j, \quad M_j = \left(1 - \frac{i\omega\chi_j}{c_j^2}\right)\gamma_j, \quad N_j = \frac{i\chi_j}{\omega}$$

γ_j is the ratio of the specific heat capacities of the liquid at constant pressure and constant volume, and χ_j is the thermal diffusivity of the liquid.

The velocity potentials of the waves passing through the layer and reflected from it will be sought in the form

$$\begin{aligned}\Psi_1 &= A_1\exp\{i[k_{11}^1x_1 + k_{11}^3(x_3 - H)]\}\\ \Psi_s &= A_2\exp\{i[k_{21}^1x_1 - k_{21}^3(x_3 + H)]\}\\ \Phi_j &= B_j\exp\{i[k_{j2}^1x_1 - (-1)^jk_{j2}^3(x_3 + (-1)^jH)]\}, \quad j = 1, 2\end{aligned} \tag{2.7}$$

where $k_{jl}^i(j, l = 1, 2; i = 1, 3)$ are the projections of the wave vector $\mathbf{k}_{jl}$ onto the x_i axis and $(k_{jl}^1)^2 + (k_{jl}^3)^2 = k_{jl}^2$. By Snell's law $k_{11}^1 = k_{12}^1 = k_{21}^1 = k_{22}^1$.

The coefficients A_j and B_j ($j = 1, 2$) are to be determined from the boundary conditions, which consist of the fact that the normal velocities of the particles of the thermoelastic medium and of the liquid are equal on both surfaces of the plane layer, the fact that there are no shear stresses on these surfaces, the normal stress and the acoustic pressure are equal on these surfaces, and the acoustic temperature and heat flux are continuous on the surfaces of the layer:

$$\begin{aligned}&x_3 = (-1)^{j-1}H\colon -i\omega u_3 = v_{jn}, \quad \sigma_{13} = 0, \quad \sigma_{33} = -p_j\\ &T = T_j, \quad \lambda_T\frac{\partial T}{\partial x_3} = \lambda_j\frac{\partial T_j}{\partial x_3}; \quad j = 1, 2\end{aligned} \tag{2.8}$$

Here

$$v_{jn} = \frac{\partial(\Psi_j + \Phi_j)}{\partial x_3}, \quad p_j = i\omega\rho_j(\Psi_j + \Phi_j)$$

$$T_j = \frac{1}{\alpha_j}\left[\frac{i\omega\gamma_j}{c_j^2}(\Psi_j + \Phi_j) + \frac{i}{\omega}\Delta(\Psi_j + \Phi_j)\right]; \quad j = 1, 2$$

where v_{jn} are the normal components of the velocities of the liquid particles, p_j are the acoustic pressures, T_j are the acoustic temperatures, and α_j and λ_j are the coefficient of thermal expansion and the thermal conductivity in the lower half-space ($j = 1$) and upper half-space ($j = 2$) respectively.

Substituting expressions (1.1), (2.3), (2.4) and (2.7) into boundary conditions (2.8), we obtain a system of nine equations, from which we find expressions for the coefficients A_j and B_j ($j = 1, 2$)

$$\mathbf{X}_j = E_j\mathbf{Y}|_{x = (-1)^{(j-1)}}, \quad j = 1, 2 \tag{2.9}$$

and six conditions for finding the particular solution of the system of differential Eq. (2.5)

$$(A\mathbf{F}' + G_j\mathbf{F})|_{x = (-1)^{j-1}} = \mathbf{D}_j, \quad j = 1, 2 \tag{2.10}$$

where

$$\mathbf{X}_j = (A_j, B_j)^T, \quad \mathbf{Y} = (U_3^*, T^*, A_i)^T, \quad \mathbf{D}_j = (0, d_2\delta_{j2}, d_3\delta_{j2})^T$$

$$E_j = \left\|\begin{matrix} e_{11}^j & e_{12}^j & e_{13}^2\delta_{j2}\\ e_{21}^j & e_{22}^j & e_{23}^2\delta_{j2}\end{matrix}\right\|, \quad G_j = \|g_{\alpha\beta}^j\|, \quad \alpha, \beta = 1, 2, 3$$

Here

$$d_2 = -\frac{i\omega\rho_2}{\mu_0}(1 + e_{13}^2 + e_{23}^2)A_i, \quad d_3 = -z_2[\xi_{21}k_{21}^3(1 - e_{13}^2) - \xi_{22}k_{22}^3 e_{23}^2]A_i$$

$$e_{11}^j = (-1)^j \frac{\xi_{j2}\omega H}{w_j}, \quad e_{12}^j = \frac{ik_{j2}^3\alpha_j T_0}{w_j}, \quad e_{13}^2 = \frac{\xi_{22}k_{21}^3 + \xi_{21}k_{22}^3}{w_2}$$

$$e_{21}^j = (-1)^{j-1}\frac{\xi_{j1}\omega H}{w_j}, \quad e_{22}^j = -\frac{ik_{j1}^3\alpha_j T_0}{w_j}, \quad e_{23}^2 = -\frac{2\xi_{21}k_{21}^3}{w_2}$$

$$w_j = \xi_{j2}k_{j1}^3 - \xi_{j1}k_{j2}^3, \quad z_j = \frac{\lambda_j H}{\lambda_T^0 \alpha_j T_0}, \quad \xi_{jl} = \frac{\omega\gamma_j}{c_j^2} - \frac{k_{jl}^2}{\omega}, \quad l = 1, 2$$

$$g_{11}^j = g_{13}^j = g_{31}^j = 0, \quad g_{12}^j = s_1\mu^*$$

$$g_{21}^j = s_1 l\lambda^*, \quad g_{2\gamma}^j = \frac{i\omega\rho_j}{\mu_0}(e_{1,\gamma-1}^j + e_{2,\gamma-1}^j) - l_1\beta^*\delta_{\gamma 3}$$

$$g_{3\gamma}^j = (-1)^{j-1}z_j(\xi_{j2}k_{j2}^3 e_{2,\gamma-1}^j + \xi_{j1}k_{j1}^3 e_{1,\gamma-1}^j), \quad \gamma = 2, 3$$

It follows from system (2.9) that the coefficients A_j and B_j can only be calculated after determining the values of the functions $U_3^*(x)$, $T^*(x)$ on the layer surfaces.

3. The solution of the boundary-value problem by the spline-collocation method

To find the functions U_3^*, T^* it is necessary to solve boundary-value problem (2.5), (2.10). We will find the solution of this problem by the spline-collocation method.[8] In the section [−1,1] we will introduce a uniform grid $-1 = x_0 < x_1 < \ldots < x_N = 1$ with a width h. We will seek an approximate solution of the boundary-value problem in the form of cubic splines $S_1(x)$, $S_2(x)$ and $S_3(x)$ of defect 1 with nodes on the grid. Here S_1, S_2 and S_3 are spline-functions, which approximate the functions U_1^*, U_3^*, T^* respectively.

We will represent the cubic splines in the form of an expansion in a basis of normalized cubic B-splines[8]

$$S_i(x) = \sum_{k=-1}^{N+1} b_k^i B_k(x), \quad i = 1, 2, 3 \tag{3.1}$$

where b_k^i are the expansion coefficients, which are to be determined, and $B_k(x)$ is a basis spline-function, defined in the interval-carrier with middle node x_k. In order that all the basis functions in (3.1) should be defined, the grid must be supplemented with the nodes

$$x_{j-3} = x_0 + (j-3)h, \quad x_{N+3-j} = x_N + (3-j)h, \quad j = 0, 1, 2$$

We will require that the splines $S_i(x)$ should satisfy system (2.5) and boundary conditions (2.10) at the collocation nodes, which coincide with the nodes of the grid. Using the expressions for the nodal values of the B-spline and its derivatives,[8] we obtain the following system of linear algebraic equations in the unknown expansion coefficients

$$\begin{gathered} P_0\mathbf{b}_0 = \mathbf{S}_0, \quad Q_k\mathbf{b}_k = 0, \quad k = 0, 1, \ldots, N, \quad R_N\mathbf{b}_N = 0 \\ \mathbf{b}_k = (b_{k-1}^1, b_{k-1}^2, b_{k-1}^3, b_k^1, b_k^2, b_k^3, b_{k+1}^1, b_{k+1}^2, b_{k+1}^3)^T \end{gathered} \tag{3.2}$$

where $\mathbf{S}_0$ is a vector of three components, the first of which is equal to zero, and P_0, Q_k and R_N are 3×9 matrices.

Solving system (3.2), consisting of $3N+9$ equations in $3N+9$ unknown coefficients, and substituting the values obtained into expressions (3.1), we obtain the approximate solution of the boundary-value problem.

By determining the coefficients for the reflected and transmitted waves from expressions (2.9), we obtain an analytical description of the wave fields outside the thermoelastic layer from formulae (2.7).

4. Solution of the boundary-value problem by the power-series method

We will construct an approximate analytic solution of boundary-value problem (2.5), (2.10) using the power-series method.[9] Here it is necessary to satisfy the requirements that, in the section $[-1,1]$ the functions $\rho^*(x)$ and $c_\upsilon^*(x)$ should be continuous with its derivatives, while the functions $\lambda^*(x)$, $\mu^*(x)$, $\alpha_T^*(x)$, $\lambda_T^*(x)$ should be continuous and have continuous derivatives up to the second order inclusive. We will assume that all these functions, and also the function $\beta^*(x)$, have the form of polynomials in x (or can be approximated by such polynomials)

$$\rho^*(x) = \sum_{m=0}^{R} \rho^{(m)} x^m, \quad \lambda^*(x) = \sum_{m=0}^{R} \lambda^{(m)} x^m, \quad \mu^*(x) = \sum_{m=0}^{R} \mu^{(m)} x^m$$

$$c_\upsilon^*(x) = \sum_{m=0}^{R} c_\upsilon^{(m)} x^m, \quad \beta^*(x) = \sum_{m=0}^{R} \beta^{(m)} x^m \tag{4.1}$$

$$\alpha_T^*(x) = \sum_{m=0}^{R} \alpha_T^{(m)} x^m, \quad \lambda_T^*(x) = \sum_{m=0}^{R} \lambda_T^{(m)} x^m$$

where R is the maximum power of the polynomials used.

Taking the above limitations into account, the solution of system (2.5) can be sought in the form of the following series

$$U_r^*(x) = \sum_{n=0}^{\infty} U_r^{(n)} x^n, \quad r = 1,3; \quad T^*(x) = \sum_{n=0}^{\infty} T^{(n)} x^n \tag{4.2}$$

which converge in the interval $[-1,1]$.

Substituting series (4.1) and (4.2) into system (2.5) and equating the coefficients of different powers of x to zero, we obtain equations for determining the coefficients $U_1^{(n)}$, $U_3^{(n)}$, $T^{(n)}$.

Solving the latter for $U_1^{(n+2)}$, $U_3^{(n+2)}$, $T^{(n+2)}$, we obtain

$$\mathbf{F}^{(n+2)} = -\frac{A^{(0)-1}}{(n+1)(n+2)} \sum_{m=0}^{R_1} \{(n+1-m)[(n-m)A^{(m+1)} + B^{(m)}] \times \\ \times \mathbf{F}^{(n+1-m)} + C^{(m)} \mathbf{F}^{(n-m)}\}, \quad R_1 = \min\{R, n\} \tag{4.3}$$

where

$$\mathbf{F}^{(n)} = (U_1^{(n)}, U_3^{(n)}, T^{(n)})^T$$

The elements of the matrices $A^{(m)}$, $B^{(m)}$ and $C^{(m)}$, besides

$$b_{11}^{(m)} = (m+1)\mu^{(m+1)}, \quad b_{22}^{(m)} = (m+1)(l\lambda^{(m+1)} + 2\mu^{(m+1)}), \quad b_{33}^{(m)} = (m+1)\lambda_T^{(m+1)}$$

$$c_{12}^{(m)} = s_1(m+1)\mu^{(m+1)}, \quad c_{21}^{(m)} = s_1 l(m+1)\lambda^{(m+1)}, \quad c_{23}^{(m)} = -l_1(m+1)\beta^{(m+1)}$$

are defined by formulae similar to (2.6), with the asterisk replaced by (m).

Note that when $m > R$ the coefficients $\rho^{(m)}$, $c_\upsilon^{(m)}$, $\lambda^{(m)}$, $\mu^{(m)}$, $\alpha_T^{(m)}$, $\lambda_T^{(m)}$, $\beta^{(m)}$ vanish.

The recurrence relations (4.3) enable us to calculate all the coefficients of expansions (4.2) with the exception of $\mathbf{F}^{(0)}$ and $\mathbf{F}^{(1)}$. The coefficients $\mathbf{F}^{(0)}$ and $\mathbf{F}^{(1)}$ are easily found if we use the reduction of boundary-value problem (2.5), (2.10) to Cauchy problems.

Suppose $\mathbf{F}_1, \mathbf{F}_2, \ldots, \mathbf{F}_6$ form a fundamental system of solutions of Eq. (2.5) in the interval (−1,1). We can choose as the fundamental system of solutions any six solutions of the Cauchy problem for system (2.5) with initial conditions which are linearly independent. Such initial conditions can be the following

$$\mathbf{F}'_\tau = (\delta_{1\tau}, \delta_{2\tau}, \delta_{3\tau})^T, \quad \mathbf{F}'_\tau = (\delta_{4\tau}, \delta_{5\tau}, \delta_{6\tau})^T, \quad \tau = 1, 2, \ldots, 6 \tag{4.4}$$

where τ is the order number of the Cauchy problem, and we can take the point $x = 0$ as the initial point. The solution $\mathbf{F}$ of the boundary-value problem will then be any linear combination

$$\mathbf{F} = \sum_{\tau=1}^{6} C_\tau \mathbf{F}_\tau \tag{4.5}$$

where

$$\mathbf{F}_\tau = \sum_{n=0}^{\infty} \mathbf{F}_\tau^{(n)} x^n, \quad \mathbf{F}_\tau^{(n)} = (U_{1\tau}^{(n)}, U_{3\tau}^{(n)}, T_\tau^{(n)})^T \tag{4.6}$$

From the relations (4.4) and (4.6) we obtain (for Cauchy problem with number τ)

$$\mathbf{F}_\tau^{(n)} = (\delta_{1\tau}, \delta_{2\tau}, \delta_{3\tau})^T, \quad \mathbf{F}_\tau^{(1)} = (\delta_{4\tau}, \delta_{5\tau}, \delta_{6\tau})^T, \quad \tau = 1, 2, \ldots, 6 \tag{4.7}$$

Hence, the coefficients $\mathbf{F}_\tau^{(n)}$ in expansion (4.6) can be calculated from formulae (4.3) and (4.7).

Substituting expression (4.5) into boundary conditions (2.10), we obtain a system of six linear algebraic equations in the unknown coefficients C_τ ($\tau = 1, 2, \ldots, 6$)

$$\sum_{\tau=1}^{6} C_\tau (A\mathbf{F}'_\tau + G_j \mathbf{F}_\tau)\Big|_{x=(-1)^{j-1}} = \mathbf{D}_j, \quad j = 1, 2 \tag{4.8}$$

After finding the coefficients C_τ we obtain an approximate analytic solution of boundary-value problem (2.5), (2.10) in the form (4.5).

5. Results of calculations

Using the solution obtained, we calculated the intensity transmission coefficient

$$W = \frac{\rho_1 c_2}{\rho_2 c_1}\left|\frac{A_1}{A_i}\right|^2$$

for plates in water ($\rho_1 = \rho_2 = 1000\ \mathrm{kg/m^3}$, $c_1 = c_2 = 1485$ m/s, $\alpha_1 = \alpha_2 = 2.1 \times 10^{-4}\ \mathrm{K^{-1}}$, $\lambda_1 = \lambda_2 = 0.59$ W/(m K), $\chi_1 = \chi_2 = 1.43 \times 10^{-7}\ \mathrm{m^2/s}$, $\gamma_1 = \gamma_2 = 1.006$ and $T_0 = 293$ K). The amplitude of the incident acoustic wave was assumed to be equal to unity, and the types of material of the plates was determined by the physical-mechanical characteristics, presented in Table 1.

The material of type A was similar in its physical-mechanical characteristics to metals (aluminium), while material of type B was similar to polymers (polyvinylbutyral).

Calculations were carried out both for uniform materials and for materials the dimensionless density of which varies linearly along the thickness of the layer: $\rho^*(x) = a(1.1 + x)$. Here the factor a was chosen so that the mean value of the function $\rho^*(x)$ was equal to unity along the thickness.

To estimate the effect of the thermoelasticity of the material of the plates on the transmission of sound, the calculations were also carried out for elastic plates for an isothermal process.

Table 1

Type of material	λ_0, N/m^2	μ_0, N/m^2	ρ_0, kg/m^3	c_v^0, J/m^3 K	α_T^0, 1/K	λ_T^0, W/m K
A	$5.6\cdot10^{10}$	$2.6\cdot10^{10}$	2700	$2.3\cdot10^6$	$25.5\cdot10^{-6}$	236
B	$3.9\cdot10^9$	$9.8\cdot10^8$	1070	$1.2\cdot10^6$	$2.3\cdot10^{-4}$	0.2

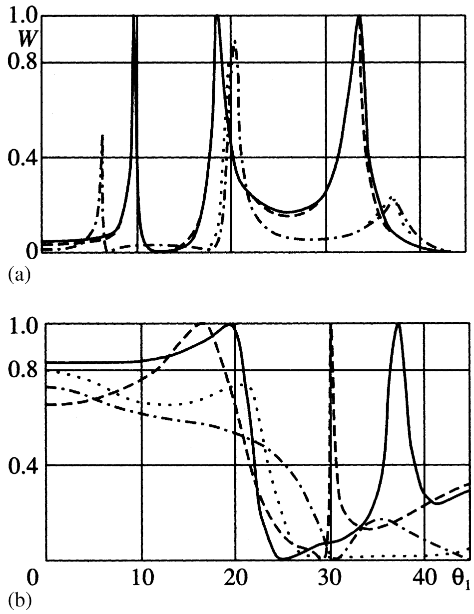

Fig. 1.

Boundary-value problem (2.5), (2.10) was solved by two methods: the spline-collocation method and the power-series method. The results of the calculations, obtained by the two methods, showed good agreement.

In Fig. 1 we show graphs of the transmission coefficient W against the angle of incidence of an acoustic wave in the range $0° \leq \theta_1 \leq 50°$ for a fixed wave dimension of the plate $2|k_{21}|H = 8.5$. Here and henceforth the continuous curves correspond to a uniform elastic material, the dashed curves correspond to a uniform thermoelastic material, the dash-dot curves correspond to a non-uniform elastic material and the dotted curves correspond to a non-uniform thermoelastic material. A comparison of the graphs in Fig. 1 for material of type A shows only a weak influence on the angular dependences of the thermoelasticity of the material. The effect of non-uniformity of the density of the material manifests itself, first, in some reduction in both the maxima of the angular characteristics in their mean level, and second, in a displacement of the positions of the maxima of the transmission coefficient. Thus, the first maximum is shifted into the region of lower angles of incidence, while the next two are shifted into the region of larger angles of incidence.

The thermoelasticity of material of type B has a considerable effect on the transmission of sound, which is illustrated by the graphs shown in the upper part of Fig. 1. This manifests itself in a clear displacement of the resonance peaks of the angular characteristics into the region of smaller angles of incidence. Here, in the case of a plate with a variable density, the resonance peak is shifted much more. An analysis of the graphs for elastic and thermoelastic plates shows that a linear variation of the density of the material smooths the angular relationships. For curves corresponding to a non-uniform material, a characteristic feature is the reduction in the global maxima by almost 20–30%. Moreover, in the range of angles of incidence investigated, only one resonance peak is observed (for an elastic layer in the region of the angle $\theta_1 = 35°$, and for a thermoelastic layer in the region of the angle $\theta_1 = 20°$). Here the height of the peak in the thermoelastic layer is approximately 4.5 times greater than in the elastic case. This feature of the angular characteristic of the transmission coefficient, constructed for a non-uniform thermoelastic material, is due to the mutual influence of the non-uniformity and the thermoelasticity of a material of type B.

In Fig. 2 we show graphs of the transmission coefficient as a function of the wave number in the range $0 < 2|k_{21}|H \leq 45$ for normal incidence of the acoustic wave. It can be seen that at low frequencies ($2|k_{21}|H < 1$) neither the thermoelasticity or the non-uniformity of the material has an effect on the transmission of the sound. As the wave number increases the effect of both thermoelasticity and non-uniformity has a greater and greater effect in displacing the resonances towards higher frequencies. The extent to which thermoelasticity of a material of type B has an effect is stronger. The graphs drawn for a uniform material show that at resonance frequencies the layer is completely transparent for the incident acoustic wave. The variable density of the material leads to some reduction in the values of the maxima of the frequency characteristics.

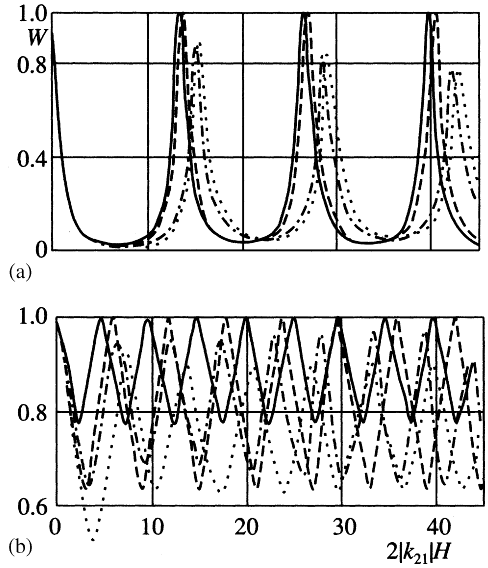

Fig. 2.

It follows from a comparison of the graphs shown in the upper part (a type A material) and the lower part (a type B material) of Fig. 2 that, in the frequency range considered, for curves corresponding to a type B material, a characteristic feature is the large number of resonance peaks and the considerable narrowing of the range of variation of the transmission coefficient. A particular feature of the effect of the thermoelasticity of a type B material on the frequency characteristics is the reduction in the minimum values of the transmission coefficient in the inter-resonance region. This manifests itself most clearly in the uniform case, where the minimum level of the coefficient W is reduced by almost 60%. A non-uniform material of this type also has a similar effect on $W(2|k_{21}|H)$. However, in the curve calculated for the case of a thermoelastic layer, the mutual effect of the non-uniformity and thermoelasticity of the material is also observed. Thus, in the range $20 \leq 2|k_{21}|H \leq 45$, the minimum values of the characteristics, corresponding to uniform and non-uniform thermoelastic materials, are almost equalized. Moreover, it can be seen from a comparison of the graphs for elastic and thermoelastic plates with a variable density that, in the thermoelastic case, the values of the local maxima of the frequency characteristic are somewhat less.

Hence, an analysis of the results of numerical calculations shows that the thermoelasticity of the material of the layer, like its non-uniformity, has a considerable influence on the transmission of sound, and the extent of this effect depends very much on the type of material.

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